

Production Inventory Model with Shortages for Deteriorating Items with Two Uncertain Rates of Deterioration

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Abstract

In this paper, a continuous production inventory model for deteriorating items with shortages is studied, in which two different rates of deterioration is considered. Production is started with one rate of deterioration; in due course of time as the production increases we have another rate of deterioration. Here the demand is considered as linear function of time, production is considered as decreasing function of time and shortages are allowed. A finite production inventory model is formulated and studied to find the effect of imperfect quality items on it. The working out of total annual inventory cost is conceded out through defuzzification process by using graded mean integration method. To illustrate the results of this proposed model, example for crisp and fuzzy sense is given

Keywords:

Deteriorating items, Rates of deterioration, Replenishment, Trapezoidal fuzzy numbers, Defuzzification, Graded mean integration method.

1. INTRODUCTION

In recent years researchers have taken keen interest in various EOQ models with different demand patterns, item shortage and more, of which deteriorating items with shortages have been focused by several researchers.

The basic assumption in any economic production order quantity model is that all produced units are of perfect quality but this is an unrealistic assumption. In any production environment imperfect items creeps in unknowingly. When the production run-time increases, probability of defective items also increases, this is due to machinery problem and improper distribution of raw materials.

Ghare P.N. and Schrader G.F [6] were the earliest researchers who developed inventory model for decaying inventory at a constant rate of deterioration. The model was extended by Covert R.P. and Philip G.C [2] with variable rate of deterioration.

Imperfect products have great impact on lot sizing policy. Chang H.C. [1] discussed the effects of imperfect items caused on the total inventory cost. Whereas Chiu S.W [4] reworked the imperfect items and framed EPQ model with random rate of deterioration. Wide research is carried out to formulate a finite production inventory model involving imperfect quality items. One among them is Goyal S.K. and Cardenas-Barron L.E [7]. Researchers Goswami and Chaudhwi [8] studied deteriorating items with shortages and linear trend in demand.

Patel R and Sheikh [10] developed an inventory model with different deterioration rates and time varying holding cost. Rosenblatt M.J. and Lee [12] incorporated the effect of defective items into the inventory problems using optimal inspection policy. Later on Salameh M.K and Jaber [13] presented an EOQ model in which demand and holding cost are considered as constant and defective items as random variables. Samanta G.P. and Roy A [14] analysed inventory model with two rates of production and back orders. Perumal and Arivarignan [11] also followed the footsteps of Samanta and Roy. Whereas Philip used three parameter Weibull distribution to represent the time of deterioration. Recently Maraiappan P, Kameswari and Antony Raj M [9] established an inventory model for deteriorating items with no shortages.

2 Methodology

2.1 Fuzzy Numbers

Any fuzzy subset of the real line R , whose membership function μ_A satisfied the following conditions, is a generalized fuzzy number \tilde{A} .

- (i) μ_A is a continuous mapping from R to the closed interval $[0, 1]$.
- (ii) $\mu_A(x) = 0, -\infty < x \leq a_1,$
- (iii) $\mu_A(x) = L(x)$ is strictly increasing on $[a_1, a_2]$
- (iv) $\mu_A(x) = w_A, a_2 \leq x \leq a_3$
- (v) $\mu_A(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$
- (vi) $\mu_A(x) = 0, a_4 \leq x < \infty$

where $0 < w_A \leq 1$ and a_1, a_2, a_3 and a_4 are real numbers. Also this type of generalized fuzzy number be denoted as $\tilde{A} = (a_1, a_2, a_3, a_4 : w_A)_{LR}$; When $w_A = 1$, it can be simplified as $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$.

2.2 Trapezoidal Fuzzy Number

A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is represented with membership function $\mu_{\tilde{A}}$ as:

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \frac{x-a}{b-a}, & \text{when } a \leq x \leq b; \\ 1 & , \text{ when } b \leq x \leq c; \\ R(x) = \frac{d-x}{d-c}, & \text{when } c \leq x \leq d; \\ 0 & , \text{ otherwise} \end{cases}$$

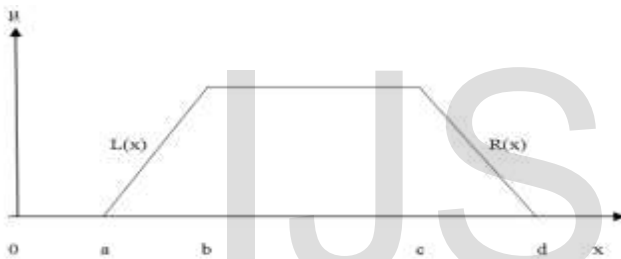


Fig.1: Trapezoidal Fuzzy Number

2.3 The Function Principle

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers, then arithmetical operations are defined as:

1. $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
2. $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$
3. $\tilde{A} \ominus \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$
4. $\tilde{A} \oslash \tilde{B} = (\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1})$
5. $\alpha \otimes \tilde{A} = \begin{cases} (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4), & \alpha \geq 0 \\ (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1), & \alpha < 0 \end{cases}$

2.4 Graded Mean Integration Method

If $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a trapezoidal fuzzy number then by the graded mean integration representation method, the defuzzified value of \tilde{A} is, $p(\tilde{A}) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$

2.5 Assumption and Notations

2.5.1 Assumption

1. Production rate is greater than demand rate and deterioration.
2. Deterioration items are either replaced or repaired during the cycle time.
3. Demand rate of the production is linear function of time $D = a+bt$.
4. Single production is considered.
5. Lead time is zero.
6. Production is decreasing function of time $P = \alpha-\beta t > D(t) > 0$.
When the inventory level reaches Q_1 the rate of deterioration is switched over to θ_2 and the production is stopped when the inventory reaches $Q_2 > Q_1$.
7. $[0, t_1]$ - No production, machines are serviced.
8. $[t_1, t_2]$ - Production starts previous shortages are met with assumption no defective items.
9. $[t_2, t_3]$ - Production at the rate of deterioration θ_1 .
10. $[t_3, t_4]$ - Production at the rate of deterioration θ_2 .
11. $[t_4, T]$ -No production only consumption by time dependent demand rate.
The cycle repeats itself after time T.

2.5.2 Notations

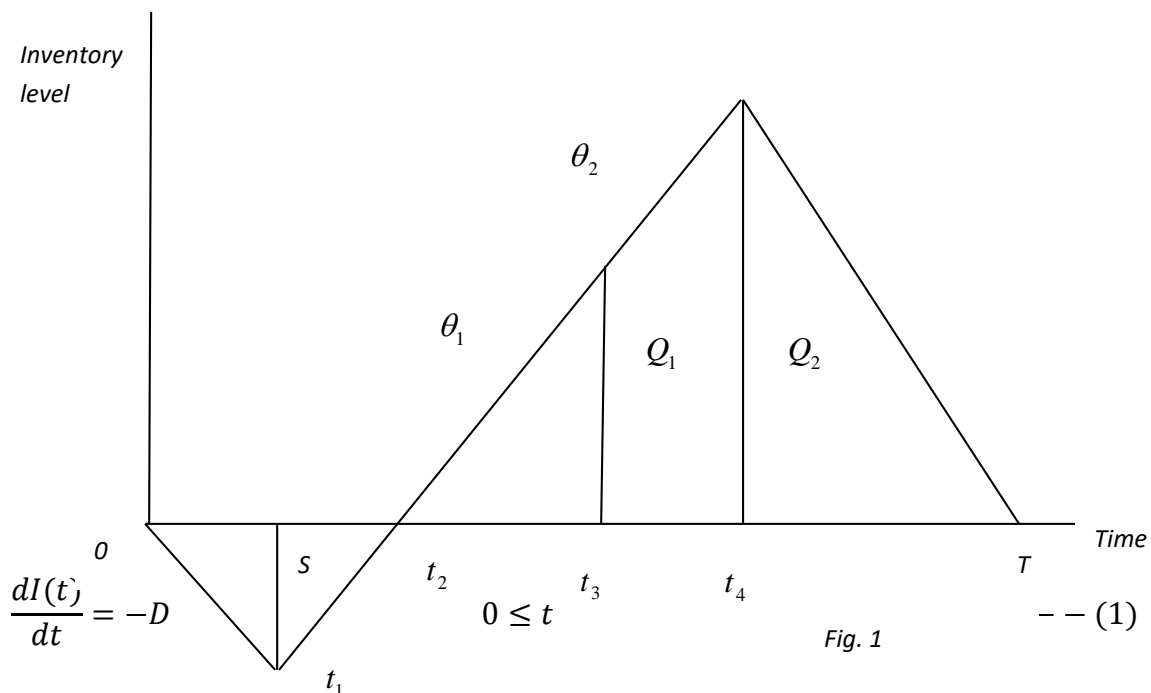
- A - The ordering cost per order
S - The shortage quantity
 Q_1 - The quantity at which the rate of deterioration is switched over to θ_2
 $D(t)$ - Demand rate at any time $t \geq 0$
T - The length of replenishment cycle
P - The purchase cost per unit item
h - The holding cost per unit per unit time
 θ_1 - The deteriorating rate during $t_2 < t < t_3$
 θ_2 - The deteriorating rate during $t_3 < t < t_4$
 $\tilde{\theta}_1$ - The fuzzy deteriorating rate during $t_2 < t < t_3$

- $\tilde{\theta}_2$ - The fuzzy deteriorating rate during $t_3 < t < t_4$
- t_1 - The time at which the inventory reaches Q_1
- t_2 - The time at which the inventory reaches zero
- t_3 - The time at which the deterioration rate is switched over to θ_2
- t_4 - The time at which the inventory level is maximum
- \tilde{t}_3 - The fuzzy time at which the deterioration rate is switched over to θ_2
- \tilde{t}_4 - The fuzzy time at which the inventory level is maximum
- Q_2 - The maximum inventory level.
- TC - The total cost of the system.

3. Model Formulation

3.1 Proposed Inventory Model in Crisp Sense:

In this section, the detailed mathematical formulation for the inventory problem is given. Here we decided to backlog demand up to S which occurs during stock out time. Thus at time t_1 , the production is started at a rate of deterioration θ_1 , when the inventory level reaches a prefixed level Q_1 , the rate of deterioration is switched over to $\theta_2 (>\theta_1)$ and continued until the inventory level reaches $Q_2 (>Q_1)$ later inventory become zero due to consumption. The behavior of the inventory model is demonstrated in fig 1. The differential equation which describes the instantaneous status of $I(t)$ over the period $(0, T)$ is given by.



$$\frac{dI(t)}{dt} = P - D \quad t_1 \leq t \leq t_2 \quad \text{----- (2)}$$

$$\frac{dI(t)}{dt} + \theta_1 I(t) = P - D \quad t_2 \leq t \leq t_3 \quad \text{----- (3)}$$

$$\frac{dI(t)}{dt} + \theta_2 I(t) = P - D \quad t_3 \leq t \leq t_4 \quad \text{----- (4)}$$

$$\frac{dI(t)}{dt} = -D \quad t_4 \leq t \leq T \quad \text{----- (5)}$$

Using the boundary conditions at $I(0) = 0$, $I(t_1) = -S$, $I(t_2) = 0$, $I(t_3) = Q_1$, $I(t_4) = Q_2$, $I(T) = 0$, we obtain the following equations

$$I(t) = -\left(at + \frac{bt^2}{2}\right) \quad 0 \leq t \leq t_1 \quad \text{----- (6)}$$

$$I(t) = (\alpha - a)(t - t_1) + \frac{(\beta - b)}{2}(t^2 - t_1^2) \quad \text{----- (7)}$$

$$I(t) = \left[\frac{(\alpha - a)}{\theta_1} + (\beta - b)\left(t - \frac{1}{\theta_1}\right)\right] + \frac{c}{e^{\theta_1 t}} \quad \text{----- (8)}$$

$$I(t) = \left(\frac{\alpha - a}{\theta_1} - \frac{\beta - b}{\theta_1}\right)(1 - e^{-\theta_1 t}) + (\beta - b)t \quad \text{----- (9)}$$

$$I(t) = \left(\frac{\alpha - a}{\theta_2} - \frac{\beta - b}{\theta_2}\right)(1 - e^{-\theta_2 t}) + (\beta - b)t \quad \text{----- (10)}$$

$$I(t) = a(t_4 - t) + \frac{b}{2}(t_4^2 - t^2) \quad \text{----- (11)}$$

From (1) & (6) put $t = t_1$

$$S = \left(at_1 + \frac{bt_1^2}{2}\right) \quad \text{----- (12)}$$

From (2) & (7) put $t = t_2$

$$(\alpha - a) + \left(\frac{\beta - b}{2}\right)(t_2 + t_1) = 0 \quad \text{----- (13)}$$

From (3) & (8) put $t = t_3$

$$Q_1 = \left(\frac{\alpha - a}{\theta_1} - \frac{\beta - b}{\theta_1}\right)(1 - e^{-\theta_1 t_3}) + (\beta - b)t_3 \quad \text{----- (14)}$$

From (4) & (10) put $t = t_4$

$$Q_2 = \left(\frac{\alpha - a}{\theta_2} - \frac{\beta - b}{\theta_2} \right) (1 - e^{-\theta_2 t_4}) + (\beta - b)t_4 \text{ --- (15)}$$

From (5) & (11) put $t = T, I(T) = 0$

$$T = t_4 + \frac{2a}{b^2} \text{ --- (16)}$$

Differentiate equation (14) w.r.to t_3 , we have

$$\frac{dQ_1}{dt_3} = - \left(\frac{\alpha - a}{\theta_1} - \frac{\beta - b}{\theta_1} \right) \frac{e^{-\theta_1 t_3}}{-\theta_1} + (\beta - b) \text{ --- (17)}$$

Now, set the equation (17) to zero and we compute for t_3 , then,

$$t_3 = \frac{(\beta - b)\theta_1}{(\alpha - a) - (\beta - b)} + \frac{1}{\theta_1} \text{ --- (18)}$$

Differentiate equation (15) w.r.to t_4 , we have

$$\frac{dQ_2}{dt_4} = - \left(\frac{\alpha - a}{\theta_2} - \frac{\beta - b}{\theta_2} \right) \frac{e^{-\theta_2 t_4}}{-\theta_2} + (\beta - b) \text{ --- (19)}$$

Now, set the equation (19) to zero and we compute for t_4 , then,

$$t_4 = \frac{(\beta - b)\theta_2}{(\alpha - a) - (\beta - b)} + \frac{1}{\theta_2} \text{ --- (20)}$$

$$t_2 - t_1 = \frac{2(a - \alpha)}{\beta - b} - 2t_1 \text{ --- (21)}$$

$$t_3 - t_2 = \frac{(\beta - b)\theta_1}{(\alpha - a) - (\beta - b)} + \frac{1}{\theta_1} - \frac{2(a - \alpha)}{\beta - b} + t_1 \text{ --- (22)}$$

$$t_4 - t_3 = \frac{(\beta - b)(\theta_2 - \theta_1)}{(\alpha - a) - (\beta - b)} + \frac{1}{\theta_2} - \frac{1}{\theta_1} \text{ --- (23)}$$

Shortage Cost:

$$SC = -C_2 \int_0^{t_2} I(t) dt$$

$$SC = -C_2 \left[\int_0^{t_1} I(t) dt + \int_{t_1}^{t_2} I(t) dt \right]$$

$$SC = -C_2 \left[\left(\frac{\alpha}{2} - a \right) t_1^2 + \left(\frac{\alpha - a}{2} \right) t_2^2 + \left(\frac{2\beta - 3b}{6} \right) t_1^3 + \left(\frac{\beta - b}{6} \right) t_2^3 - [(\alpha - a) + (\beta - b)t_1] t_1 t_2 \right]$$

----- (24)

Deterioration cost:

$$DC = C_1 \int_{t_2}^{t_3} \theta_1 I(t) dt + \int_{t_3}^{t_4} \theta_2 I(t) dt$$

$$DC = C_1 \left\{ [(\alpha - a) - (\beta - b)] \left[t_4 - t_2 + \frac{2}{\theta_1} + \frac{2}{\theta_2} \right] + \frac{\beta - b}{2} [\theta_1 (t_3 - t_2)^2 + \theta_2 (t_4 - t_3)^2] \right\}$$

----- (25)

Purchasing cost:

$$PC = C_3 [P(t_2 - t_1) + P(t_3 - t_2) + P(t_4 - t_3)]$$

$$PC = C_3 \left\{ (\alpha - \beta t_1) \left(\frac{2(a - \alpha)}{(\beta - b)} - 2t_1 \right) + (\alpha - \beta t_2) \left(\frac{(\beta - b)\theta_1}{(\alpha - a) - (\beta - b)} + \frac{1}{\theta_1} - \frac{2(a - \alpha)}{\beta - b} + t_1 \right) + (\alpha - \beta t_3) \left(\frac{(\beta - b)(\theta_2 - \theta_1)}{(\alpha - a) - (\beta - b)} + \frac{1}{\theta_2} - \frac{1}{\theta_1} \right) \right\}$$

----- (26)

Total Inventory Cost:

$$TC = \frac{1}{T} [PC + SC + DC]$$

$$\begin{aligned}
 TC = \frac{1}{T} & \left[C_3 \left\{ (\alpha - \beta t_1) \left(\frac{2(a - \alpha)}{(\beta - b)} - 2t_1 \right) + (\alpha - \beta t_2) \left(\frac{(\beta - b)\theta_1}{(\alpha - a) - (\beta - b)} + \frac{1}{\theta_1} - \frac{2(a - \alpha)}{\beta - b} + t_1 \right) \right. \right. \\
 & + (\alpha - \beta t_3) \left(\frac{(\beta - b)(\theta_2 - \theta_1)}{(\alpha - a) - (\beta - b)} + \frac{1}{\theta_2} - \frac{1}{\theta_1} \right) \left. \right\} \\
 & + C_1 \left\{ [((\alpha - a) - (\beta - b))] \left[t_4 - t_2 + \frac{2}{\theta_1} + \frac{2}{\theta_2} \right] \right. \\
 & + \frac{\beta - b}{2} [\theta_1(t_3 - t_2)^2 + \theta_2(t_4 - t_3)^2] \left. \right\} \\
 & - C_2 \left[\left(\frac{\alpha}{2} - a \right) t_1^2 + \left(\frac{\alpha - a}{2} \right) t_2^2 + \left(\frac{2\beta - 3b}{6} \right) t_1^3 + \left(\frac{\beta - b}{6} \right) t_2^3 \right. \\
 & \left. - [(\alpha - a) + (\beta - b)t_1]t_1t_2 \right] \quad \text{----- (27)}
 \end{aligned}$$

To maximize the total cost (TC) per unit time, the optimal value of T can be obtained by solving the following equations

$$\frac{\partial TC}{\partial T} = 0$$

Provided that above equations satisfies the following conditions

$$\frac{\partial^2 TC}{\partial T^2} > 0$$

$$\begin{aligned}
 \frac{\partial TC}{\partial T} = -\frac{1}{T^2} & \left[C_3 \left\{ (\alpha - \beta t_1) \left(\frac{2(a - \alpha)}{(\beta - b)} - 2t_1 \right) \right. \right. \\
 & + (\alpha - \beta t_2) \left(\frac{(\beta - b)\theta_1}{(\alpha - a) - (\beta - b)} + \frac{1}{\theta_1} - \frac{2(a - \alpha)}{\beta - b} + t_1 \right) \\
 & + (\alpha - \beta t_3) \left(\frac{(\beta - b)(\theta_2 - \theta_1)}{(\alpha - a) - (\beta - b)} + \frac{1}{\theta_2} - \frac{1}{\theta_1} \right) \left. \right\} \\
 & + C_1 \left\{ [((\alpha - a) - (\beta - b))] \left[t_4 - t_2 + \frac{2}{\theta_1} + \frac{2}{\theta_2} \right] \right. \\
 & + \frac{\beta - b}{2} [\theta_1(t_3 - t_2)^2 + \theta_2(t_4 - t_3)^2] \left. \right\} \\
 & - C_2 \left[\left(\frac{\alpha}{2} - a \right) t_1^2 + \left(\frac{\alpha - a}{2} \right) t_2^2 + \left(\frac{2\beta - 3b}{6} \right) t_1^3 + \left(\frac{\beta - b}{6} \right) t_2^3 \right. \\
 & \left. - [(\alpha - a) + (\beta - b)t_1]t_1t_2 \right] \left. \right] = 0 \quad \text{----- (28)}
 \end{aligned}$$

$$\begin{aligned}
 C_3 \left\{ (\alpha - \beta t_1) \left(\frac{2(a - \alpha)}{(\beta - b)} - 2t_1 \right) + (\alpha - \beta t_2) \left(\frac{(\beta - b)\theta_1}{(\alpha - a) - (\beta - b)} + \frac{1}{\theta_1} - \frac{2(a - \alpha)}{\beta - b} + t_1 \right) \right. \\
 + (\alpha - \beta t_3) \left(\frac{(\beta - b)(\theta_2 - \theta_1)}{(\alpha - a) - (\beta - b)} + \frac{1}{\theta_2} - \frac{1}{\theta_1} \right) \left. \right\} \\
 + C_1 \left\{ [((\alpha - a) - (\beta - b))] \left[t_4 - t_2 + \frac{2}{\theta_1} + \frac{2}{\theta_2} \right] \right. \\
 + \frac{\beta - b}{2} [\theta_1(t_3 - t_2)^2 + \theta_2(t_4 - t_3)^2] \left. \right\} \\
 - C_2 \left[\left(\frac{\alpha}{2} - a \right) t_1^2 + \left(\frac{\alpha - a}{2} \right) t_2^2 + \left(\frac{2\beta - 3b}{6} \right) t_1^3 + \left(\frac{\beta - b}{6} \right) t_2^3 \right. \\
 \left. - [(\alpha - a) + (\beta - b)t_1]t_1t_2 \right] = 0 \quad \text{----- (29)}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial^2 TC}{\partial T^2} = \frac{2}{T^3} \left[C_3 \left\{ (\alpha - \beta t_1) \left(\frac{2(a - \alpha)}{(\beta - b)} - 2t_1 \right) \right. \right. \\
 + (\alpha - \beta t_2) \left(\frac{(\beta - b)\theta_1}{(\alpha - a) - (\beta - b)} + \frac{1}{\theta_1} - \frac{2(a - \alpha)}{\beta - b} + t_1 \right) \\
 + (\alpha - \beta t_3) \left(\frac{(\beta - b)(\theta_2 - \theta_1)}{(\alpha - a) - (\beta - b)} + \frac{1}{\theta_2} - \frac{1}{\theta_1} \right) \left. \right\} \\
 + C_1 \left\{ [((\alpha - a) - (\beta - b))] \left[t_4 - t_2 + \frac{2}{\theta_1} + \frac{2}{\theta_2} \right] \right. \\
 + \frac{\beta - b}{2} [\theta_1(t_3 - t_2)^2 + \theta_2(t_4 - t_3)^2] \left. \right\} \\
 - C_2 \left[\left(\frac{\alpha}{2} - a \right) t_1^2 + \left(\frac{\alpha - a}{2} \right) t_2^2 + \left(\frac{2\beta - 3b}{6} \right) t_1^3 + \left(\frac{\beta - b}{6} \right) t_2^3 \right. \\
 \left. - [(\alpha - a) + (\beta - b)t_1]t_1t_2 \right] \Bigg] > 0 \quad \text{----- (30)}
 \end{aligned}$$

3.2 Proposed Inventory Model in Fuzzy Sense:

$$\frac{dI(t)}{dt} = -D \quad 0 \leq t \leq t_1 \quad \text{----- (31)}$$

$$\frac{dI(t)}{dt} = P - D \quad t_1 \leq t \leq t_2 \quad \text{----- (32)}$$

$$\frac{dI(t)}{dt} + \tilde{\theta}_1 I(t) = P - D \quad t_2 \leq t \leq t_3 \quad \text{----- (33)}$$

$$\frac{dI(t)}{dt} + \tilde{\theta}_2 I(t) = P - D \quad t_3 \leq t \leq t_4 \quad \text{----- (34)}$$

$$\frac{dI(t)}{dt} = -D \quad t_4 \leq t \leq T \quad \text{----- (35)}$$

Using the boundary conditions at $I(0) = 0$, $I(t_1) = -S$, $I(t_2) = 0$, $I(t_3) = Q_1$, $I(t_4) = Q_2$, $I(T) = 0$, we obtain the following equations

$$I(t) = -\left(at + \frac{bt^2}{2}\right) \quad 0 \leq t \leq t_1 \quad \text{----- (36)}$$

$$I(t) = (\alpha - a)(t - t_1) + \frac{(\beta - b)}{2}(t^2 - t_1^2) \quad \text{----- (37)}$$

$$I(t) = \left[\frac{(\alpha - a)}{\tilde{\theta}_1} + (\beta - b)\left(t - \frac{1}{\tilde{\theta}_1}\right)\right] + \frac{c}{e^{\tilde{\theta}_1 t}} \quad \text{----- (38)}$$

$$I(t) = \left(\frac{\alpha - a}{\tilde{\theta}_1} - \frac{\beta - b}{\tilde{\theta}_1}\right)(1 - e^{-\tilde{\theta}_1 t}) + (\beta - b)t \quad \text{----- (39)}$$

$$I(t) = \left(\frac{\alpha - a}{\tilde{\theta}_2} - \frac{\beta - b}{\tilde{\theta}_2}\right)(1 - e^{-\tilde{\theta}_2 t}) + (\beta - b)t \quad \text{----- (40)}$$

$$I(t) = a(t_4 - t) + \frac{b}{2}(t_4^2 - t^2) \quad \text{----- (41)}$$

From (31) & (36) put $t = t_1$

$$S = \left(at_1 + \frac{bt_1^2}{2}\right) \quad \text{----- (42)}$$

From (32) & (37) put $t = t_2$

$$(\alpha - a) + \left(\frac{\beta - b}{2}\right)(t_2 + t_1) = 0 \quad \text{----- (43)}$$

From (33) & (38) put $t = t_3$

$$Q_1 = \left(\frac{\alpha - a}{\tilde{\theta}_1} - \frac{\beta - b}{\tilde{\theta}_1}\right)(1 - e^{-\tilde{\theta}_1 t_3}) + (\beta - b)t_3 \quad \text{----- (44)}$$

From (34) & (40) put $t = t_4$

$$Q_2 = \left(\frac{\alpha - a}{\tilde{\theta}_2} - \frac{\beta - b}{\tilde{\theta}_2}\right)(1 - e^{-\tilde{\theta}_2 t_4}) + (\beta - b)t_4 \quad \text{----- (45)}$$

From (35) & (41) put $t = T$, $I(T) = 0$

$$T = t_4 + \frac{2a}{b^2} \quad \text{----- (46)}$$

Differentiate equation (44) w.r.to t_3 , we have

$$\frac{dQ_1}{dt_3} = -\left(\frac{\alpha - a}{\tilde{\theta}_1} - \frac{\beta - b}{\tilde{\theta}_1}\right) \frac{e^{-\tilde{\theta}_1 t_3}}{-\tilde{\theta}_1} + (\beta - b) \dots \dots \dots (47)$$

Now, set the equation (47) to zero and we compute for t_3 , then,

$$t_3 = \frac{(\beta - b)\tilde{\theta}_1}{(\alpha - a) - (\beta - b)} + \frac{1}{\tilde{\theta}_1} \dots \dots \dots (48)$$

Differentiate equation (45) w.r.to t_4 , we have

$$\frac{dQ_2}{dt_4} = -\left(\frac{\alpha - a}{\tilde{\theta}_2} - \frac{\beta - b}{\tilde{\theta}_2}\right) \frac{e^{-\tilde{\theta}_2 t_4}}{-\tilde{\theta}_2} + (\beta - b) \dots \dots \dots (49)$$

Now, set the equation (49) to zero and we compute for t_4 , then,

$$t_4 = \frac{(\beta - b)\tilde{\theta}_2}{(\alpha - a) - (\beta - b)} + \frac{1}{\tilde{\theta}_2} \dots \dots \dots (50)$$

$$t_2 - t_1 = \frac{2(a - \alpha)}{\beta - b} - 2 t_1 \dots \dots \dots (51)$$

$$t_3 - t_2 = \frac{(\beta - b)\tilde{\theta}_1}{(\alpha - a) - (\beta - b)} + \frac{1}{\tilde{\theta}_1} - \frac{2(a - \alpha)}{\beta - b} + t_1 \dots \dots \dots (52)$$

$$t_4 - t_3 = \frac{(\beta - b)(\tilde{\theta}_2 - \tilde{\theta}_1)}{(\alpha - a) - (\beta - b)} + \frac{1}{\tilde{\theta}_2} - \frac{1}{\tilde{\theta}_1} \dots \dots \dots (53)$$

Shortage Cost:

$$SC = -C_2 \int_0^{t_2} I(t)dt$$

$$SC = -C_2 \left[\int_0^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt \right]$$

$$SC = -C_2 \left[\left(\frac{\alpha}{2} - a\right) t_1^2 + \left(\frac{\alpha - a}{2}\right) t_2^2 + \left(\frac{2\beta - 3b}{6}\right) t_1^3 + \left(\frac{\beta - b}{6}\right) t_2^3 - [(\alpha - a) + (\beta - b)t_1]t_1 t_2 \right] \dots \dots \dots (54)$$

Deterioration cost:

$$DC = C_1 \int_{t_2}^{t_3} \tilde{\theta}_1 I(t) dt + \int_{t_3}^{t_4} \tilde{\theta}_2 I(t) dt$$

$$DC = C_1 \left\{ [((\alpha - a) - (\beta - b))] \left[t_4 - t_2 + \frac{2}{\tilde{\theta}_1} + \frac{2}{\tilde{\theta}_2} \right] + \frac{\beta - b}{2} [\tilde{\theta}_1 (t_3 - t_2)^2 + \tilde{\theta}_2 (t_4 - t_3)^2] \right\} \quad (55)$$

Purchasing cost:

$$PC = C_3 [P(t_2 - t_1) + P(t_3 - t_2) + P(t_4 - t_3)]$$

$$PC = C_3 \left\{ (\alpha - \beta t_1) \left(\frac{2(a - \alpha)}{(\beta - b)} - 2t_1 \right) + (\alpha - \beta t_2) \left(\frac{(\beta - b)\tilde{\theta}_1}{(\alpha - a) - (\beta - b)} + \frac{1}{\tilde{\theta}_1} - \frac{2(a - \alpha)}{\beta - b} + t_1 \right) \right.$$

$$\left. + (\alpha - \beta t_3) \left(\frac{(\beta - b)(\tilde{\theta}_2 - \tilde{\theta}_1)}{(\alpha - a) - (\beta - b)} + \frac{1}{\tilde{\theta}_2} - \frac{1}{\tilde{\theta}_1} \right) \right\} \quad (56)$$

Total Inventory Cost:

$$TC = \frac{1}{T} [PC + SC + DC]$$

$$TC = \frac{1}{T} \left[C_3 \left\{ (\alpha - \beta t_1) \left(\frac{2(a - \alpha)}{(\beta - b)} - 2t_1 \right) + (\alpha - \beta t_2) \left(\frac{(\beta - b)\tilde{\theta}_1}{(\alpha - a) - (\beta - b)} + \frac{1}{\tilde{\theta}_1} - \frac{2(a - \alpha)}{\beta - b} + t_1 \right) \right. \right.$$

$$\left. + (\alpha - \beta t_3) \left(\frac{(\beta - b)(\tilde{\theta}_2 - \tilde{\theta}_1)}{(\alpha - a) - (\beta - b)} + \frac{1}{\tilde{\theta}_2} - \frac{1}{\tilde{\theta}_1} \right) \right\}$$

$$+ C_1 \left\{ [((\alpha - a) - (\beta - b))] \left[t_4 - t_2 + \frac{2}{\tilde{\theta}_1} + \frac{2}{\tilde{\theta}_2} \right] \right.$$

$$\left. + \frac{\beta - b}{2} [\tilde{\theta}_1 (t_3 - t_2)^2 + \tilde{\theta}_2 (t_4 - t_3)^2] \right\}$$

$$- C_2 \left[\left(\frac{\alpha}{2} - a \right) t_1^2 + \left(\frac{\alpha - a}{2} \right) t_2^2 + \left(\frac{2\beta - 3b}{6} \right) t_1^3 + \left(\frac{\beta - b}{6} \right) t_2^3 \right.$$

$$\left. - [(\alpha - a) + (\beta - b)t_1] t_1 t_2 \right] \quad (57)$$

To maximize the total cost (TC) per unit time, the optimal value of T can be obtained by solving the following equation

$$\frac{\partial TC}{\partial T} = 0$$

Provided that above equation satisfies the following conditions

$$\frac{\partial^2 TC}{\partial T^2} > 0$$

$$\begin{aligned} \frac{\partial TC}{\partial T} = & -\frac{1}{T^2} \left[C_3 \left\{ (\alpha - \beta t_1) \left(\frac{2(a - \alpha)}{(\beta - b)} - 2t_1 \right) \right. \right. \\ & + (\alpha - \beta t_2) \left(\frac{(\beta - b)\tilde{\theta}_1}{(\alpha - a) - (\beta - b)} + \frac{1}{\tilde{\theta}_1} - \frac{2(a - \alpha)}{\beta - b} + t_1 \right) \\ & + (\alpha - \beta t_3) \left(\frac{(\beta - b)(\tilde{\theta}_2 - \tilde{\theta}_1)}{(\alpha - a) - (\beta - b)} + \frac{1}{\tilde{\theta}_2} - \frac{1}{\tilde{\theta}_1} \right) \left. \right\} \\ & + C_1 \left\{ [((\alpha - a) - (\beta - b))] \left[t_4 - t_2 + \frac{2}{\tilde{\theta}_1} + \frac{2}{\tilde{\theta}_2} \right] \right. \\ & + \frac{\beta - b}{2} [\tilde{\theta}_1(t_3 - t_2)^2 + \tilde{\theta}_2(t_4 - t_3)^2] \left. \right\} \\ & - C_2 \left[\left(\frac{\alpha}{2} - a \right) t_1^2 + \left(\frac{\alpha - a}{2} \right) t_2^2 + \left(\frac{2\beta - 3b}{6} \right) t_1^3 + \left(\frac{\beta - b}{6} \right) t_2^3 \right. \\ & \left. - [(\alpha - a) + (\beta - b)t_1]t_1t_2 \right] = 0 \end{aligned} \quad \text{--- (58)}$$

$$\begin{aligned} C_3 \left\{ (\alpha - \beta t_1) \left(\frac{2(a - \alpha)}{(\beta - b)} - 2t_1 \right) + (\alpha - \beta t_2) \left(\frac{(\beta - b)\tilde{\theta}_1}{(\alpha - a) - (\beta - b)} + \frac{1}{\tilde{\theta}_1} - \frac{2(a - \alpha)}{\beta - b} + t_1 \right) \right. \\ + (\alpha - \beta t_3) \left(\frac{(\beta - b)(\tilde{\theta}_2 - \tilde{\theta}_1)}{(\alpha - a) - (\beta - b)} + \frac{1}{\tilde{\theta}_2} - \frac{1}{\tilde{\theta}_1} \right) \left. \right\} \\ + C_1 \left\{ [((\alpha - a) - (\beta - b))] \left[t_4 - t_2 + \frac{2}{\tilde{\theta}_1} + \frac{2}{\tilde{\theta}_2} \right] \right. \\ + \frac{\beta - b}{2} [\tilde{\theta}_1(t_3 - t_2)^2 + \tilde{\theta}_2(t_4 - t_3)^2] \left. \right\} \\ - C_2 \left[\left(\frac{\alpha}{2} - a \right) t_1^2 + \left(\frac{\alpha - a}{2} \right) t_2^2 + \left(\frac{2\beta - 3b}{6} \right) t_1^3 + \left(\frac{\beta - b}{6} \right) t_2^3 \right. \\ \left. - [(\alpha - a) + (\beta - b)t_1]t_1t_2 \right] = 0 \end{aligned} \quad \text{--- (59)}$$

$$\begin{aligned} \frac{\partial^2 TC}{\partial T^2} = \frac{2}{T^3} & \left[C_3 \left\{ (\alpha - \beta t_1) \left(\frac{2(a - \alpha)}{(\beta - b)} - 2t_1 \right) \right. \right. \\ & + (\alpha - \beta t_2) \left(\frac{(\beta - b)\tilde{\theta}_1}{(\alpha - a) - (\beta - b)} + \frac{1}{\tilde{\theta}_1} - \frac{2(a - \alpha)}{\beta - b} + t_1 \right) \\ & + (\alpha - \beta t_3) \left(\frac{(\beta - b)(\tilde{\theta}_2 - \tilde{\theta}_1)}{(\alpha - a) - (\beta - b)} + \frac{1}{\tilde{\theta}_2} - \frac{1}{\tilde{\theta}_1} \right) \left. \right\} \\ & + C_1 \left\{ [((\alpha - a) - (\beta - b))] \left[t_4 - t_2 + \frac{2}{\tilde{\theta}_1} + \frac{2}{\tilde{\theta}_2} \right] \right. \\ & + \frac{\beta - b}{2} [\tilde{\theta}_1(t_3 - t_2)^2 + \tilde{\theta}_2(t_4 - t_3)^2] \left. \right\} \\ & - C_2 \left[\left(\frac{\alpha}{2} - a \right) t_1^2 + \left(\frac{\alpha - a}{2} \right) t_2^2 + \left(\frac{2\beta - 3b}{6} \right) t_1^3 + \left(\frac{\beta - b}{6} \right) t_2^3 \right. \\ & \left. \left. - [(\alpha - a) + (\beta - b)t_1]t_1t_2 \right] \right] > 0 \end{aligned} \quad \text{--- (60)}$$

4. Numerical Illustration:

4.1 Numerical Illustration for Crisp sense:

Let $\theta_1 = 0.2$, $\theta_2 = 0.1$, $c_1 = 2$, $c_2 = 3$, $c_3 = 20$, $\alpha = 2$, $\beta = 1$, $a = 1$, $b = 0.5$, $t_1 = 2$, $t_2 = 4$. Length of replenishment cycle, shortage quantity, quantity which the rate of deterioration, maximum inventory level, shortage cost, deterioration cost, purchasing cost and total cost are determined.

Solution:

$$t_3 = \frac{(\beta - b)\theta_1}{(\alpha - a) - (\beta - b)} + \frac{1}{\theta_1} = 5 \text{ days}$$

$$t_4 = \frac{(\beta - b)\theta_2}{(\alpha - a) - (\beta - b)} + \frac{1}{\theta_2} = 10 \text{ days}$$

4.1.1 Length of replenishment cycle:

$$T = t_4 + \frac{2a}{b^2} = 18 \text{ days}$$

4.1.2 Shortage quantity:

$$S = \left(at_1 + \frac{bt_1^2}{2} \right) = 3$$

4.1.3 Quantity which the rate of deterioration is switched over to θ_2 :

$$Q_1 = \left(\frac{\alpha-a}{\theta_1} - \frac{\beta-b}{\theta_1}\right)(1 - e^{-\theta_1 t_3}) + (\beta - b)t_3 = 4.0803$$

4.1.4 Maximum inventory level:

$$Q_2 = \left(\frac{\alpha-a}{\theta_2} - \frac{\beta-b}{\theta_2}\right)(1 - e^{-\theta_2 t_4}) + (\beta - b)t_4 = 8.1605$$

4.1.5 Shortage cost:

$$SC = -C_2 \left[\left(\frac{\alpha}{2} - a\right) t_1^2 + \left(\frac{\alpha-a}{2}\right) t_2^2 + \left(\frac{2\beta-3b}{6}\right) t_1^3 + \left(\frac{\beta-b}{6}\right) t_2^3 - [(\alpha - a) + (\beta - b)t_1]t_1 t_2 \right] = 6$$

4.1.6 Deterioration cost:

$$DC = C_1 \left\{ [((\alpha - a) - (\beta - b))] \left[t_4 - t_2 + \frac{2}{\theta_1} + \frac{2}{\theta_2} \right] + \frac{\beta - b}{2} [\theta_1(t_3 - t_2)^2 + \theta_2(t_4 - t_3)^2] \right\} \\ = Rs. 37.35$$

4.1.7 Purchasing cost:

$$PC = C_3 \left\{ (\alpha - \beta t_1) \left(\frac{2(a - \alpha)}{(\beta - b)} - 2t_1 \right) + (\alpha - \beta t_2) \left(\frac{(\beta - b)\theta_1}{(\alpha - a) - (\beta - b)} + \frac{1}{\theta_1} - \frac{2(a - \alpha)}{\beta - b} + t_1 \right) \right. \\ \left. + (\alpha - \beta t_3) \left(\frac{(\beta - b)(\theta_2 - \theta_1)}{(\alpha - a) - (\beta - b)} + \frac{1}{\theta_2} - \frac{1}{\theta_1} \right) \right\} = Rs. 146$$

4.1.8 Total inventory cost:

$$TC = \frac{1}{T} \left[C_3 \left\{ (\alpha - \beta t_1) \left(\frac{2(a - \alpha)}{(\beta - b)} - 2t_1 \right) + (\alpha - \beta t_2) \left(\frac{(\beta - b)\theta_1}{(\alpha - a) - (\beta - b)} + \frac{1}{\theta_1} - \frac{2(a - \alpha)}{\beta - b} + t_1 \right) \right. \right. \\ \left. \left. + (\alpha - \beta t_3) \left(\frac{(\beta - b)(\theta_2 - \theta_1)}{(\alpha - a) - (\beta - b)} + \frac{1}{\theta_2} - \frac{1}{\theta_1} \right) \right\} \right. \\ \left. + C_1 \left\{ [((\alpha - a) - (\beta - b))] \left[t_4 - t_2 + \frac{2}{\theta_1} + \frac{2}{\theta_2} \right] \right. \right. \\ \left. \left. + \frac{\beta - b}{2} [\theta_1(t_3 - t_2)^2 + \theta_2(t_4 - t_3)^2] \right\} \right. \\ \left. - C_2 \left[\left(\frac{\alpha}{2} - a\right) t_1^2 + \left(\frac{\alpha-a}{2}\right) t_2^2 + \left(\frac{2\beta-3b}{6}\right) t_1^3 + \left(\frac{\beta-b}{6}\right) t_2^3 \right. \right. \\ \left. \left. - [(\alpha - a) + (\beta - b)t_1]t_1 t_2 \right] \right]$$

= Rs. 189.35

4.2 Numerical Illustration for fuzzy sense:

Let $\tilde{\theta}_1 = (0.1, 0.2, 0.3, 0.4)$, $\tilde{\theta}_2 = (0.05, 0.1, 0.15, 0.2)$, $c_1 = 2, c_2 = 3, c_3 = 20, \alpha = 2, \beta = 1, a = 1,$
 $b = 0.5, t_1 = 2, t_2 = 4$. Length of replenishment cycle, shortage quantity, quantity which the rate of
 deterioration, maximum inventory level, shortage cost, deterioration cost, purchasing cost, and
 total cost are determined.

Solution:

$$\tilde{t}_3 = \frac{(\beta-b)\tilde{\theta}_1}{(\alpha-a)-(\beta-b)} + \frac{1}{\tilde{\theta}_1} = (2.6, 3.53, 5.3, 10.4)$$

Graded mean integration method:

$$p(\tilde{t}_3) \approx 5 \text{ days}$$

$$\tilde{t}_4 = \frac{(\beta-b)\tilde{\theta}_2}{(\alpha-a)-(\beta-b)} + \frac{1}{\tilde{\theta}_2} = (5.05, 6.7667, 10.15, 20.2)$$

Graded mean integration method:

$$p(\tilde{t}_4) \approx 10 \text{ days}$$

4.2.1 Length of replenishment cycle:

$$\tilde{T} = \tilde{t}_4 + \frac{2a}{b^2} = (13.05, 14.7667, 18.15, 28.20) \text{ days}$$

Graded mean integration method:

$$p(\tilde{T}) \approx 18 \text{ days}$$

4.2.2 Shortage quantity:

$$S = \left(at_1 + \frac{bt_1^2}{2} \right) = 3$$

4.2.3 Quantity which the rate of deterioration is switched over to θ_2 :

$$\tilde{Q}_1 = \left(\frac{\alpha-a}{\tilde{\theta}_1} - \frac{\beta-b}{\tilde{\theta}_1} \right) (1 - e^{-\tilde{\theta}_1 t_3}) + (\beta - b)t_3 = (3.0733, 3.6089, 4.3869, 6.7418)$$

Graded mean integration method:

$$p(\tilde{Q}_1) \approx 4.3011$$

4.2.4 Maximum inventory level:

$$\tilde{Q}_2 = \left(\frac{\alpha-a}{\tilde{\theta}_2} - \frac{\beta-b}{\tilde{\theta}_2} \right) (1 - e^{-\tilde{\theta}_2 t_4}) + (\beta - b)t_4 = (6.1467, 7.2176, 8.7737, 13.4837)$$

Graded mean integration method:

$$p(\tilde{Q}_2) = 8.6022$$

4.2.5 Shortage Cost:

$$SC = -C_2 \left[\left(\frac{\alpha}{2} - a \right) t_1^2 + \left(\frac{\alpha-a}{2} \right) t_2^2 + \left(\frac{2\beta-3b}{6} \right) t_1^3 + \left(\frac{\beta-b}{6} \right) t_2^3 - [(\alpha - a) + (\beta - b)t_1]t_1t_2 \right] = 6$$

4.2.6 Deterioration cost:

$$\begin{aligned} \tilde{DC} &= C_1 \left\{ [((\alpha - a) - (\beta - b))] \left[t_4 - t_2 + \frac{2}{\tilde{\theta}_1} + \frac{2}{\tilde{\theta}_2} \right] + \frac{\beta - b}{2} [\tilde{\theta}_1(t_3 - t_2)^2 + \tilde{\theta}_2(t_4 - t_3)^2] \right\} \\ &= (21.675, 27.35, 38.025, 68.7) \end{aligned}$$

Graded mean integration method:

$$p(\tilde{DC}) = 8.6022$$

4.2.7 Purchasing cost:

$$\begin{aligned} \tilde{PC} &= C_3 \left\{ (\alpha - \beta t_1) \left(\frac{2(a - \alpha)}{(\beta - b)} - 2t_1 \right) + (\alpha - \beta t_2) \left(\frac{(\beta - b)\tilde{\theta}_1}{(\alpha - a) - (\beta - b)} + \frac{1}{\tilde{\theta}_1} - \frac{2(a - \alpha)}{\beta - b} + t_1 \right) \right. \\ &\quad \left. + (\alpha - \beta t_3) \left(\frac{(\beta - b)(\tilde{\theta}_2 - \tilde{\theta}_1)}{(\alpha - a) - (\beta - b)} + \frac{1}{\tilde{\theta}_2} - \frac{1}{\tilde{\theta}_1} \right) \right\} \end{aligned}$$

$$= (134.7921, 140.3545, 152.7396, 158.6301)$$

Graded mean integration method:

$$p(\tilde{PC}) = 146.6$$

4.2.8 Total annual inventory cost:

$$\begin{aligned}
 TC = \frac{1}{T} & \left[C_3 \left\{ (\alpha - \beta t_1) \left(\frac{2(a - \alpha)}{(\beta - b)} - 2t_1 \right) + (\alpha - \beta t_2) \left(\frac{(\beta - b)\tilde{\theta}_1}{(\alpha - a) - (\beta - b)} + \frac{1}{\tilde{\theta}_1} - \frac{2(a - \alpha)}{\beta - b} + t_1 \right) \right. \right. \\
 & + (\alpha - \beta t_3) \left(\frac{(\beta - b)(\tilde{\theta}_2 - \tilde{\theta}_1)}{(\alpha - a) - (\beta - b)} + \frac{1}{\tilde{\theta}_2} - \frac{1}{\tilde{\theta}_1} \right) \left. \right\} \\
 & + C_1 \left\{ [((\alpha - a) - (\beta - b))] \left[t_4 - t_2 + \frac{2}{\tilde{\theta}_1} + \frac{2}{\tilde{\theta}_2} \right] \right. \\
 & + \left. \frac{\beta - b}{2} [\tilde{\theta}_1(t_3 - t_2)^2 + \tilde{\theta}_2(t_4 - t_3)^2] \right\} \\
 & - C_2 \left[\left(\frac{\alpha}{2} - a \right) t_1^2 + \left(\frac{\alpha - a}{2} \right) t_2^2 + \left(\frac{2\beta - 3b}{6} \right) t_1^3 + \left(\frac{\beta - b}{6} \right) t_2^3 \right. \\
 & \left. \left. - [(\alpha - a) + (\beta - b)t_1]t_1t_2 \right] \right]
 \end{aligned}$$

= (177.4328, 183.7391, 195.9371, 201.5321)

Graded mean integration method:

p (TC̃) ≈ 189.7196

5. Conclusion:

In this paper, we have found an optimum total annual inventory cost in the crisp sense as well as in the fuzzy sense. Two deterioration rates are taken as trapezoidal fuzzy numbers. This model is solved analytically by minimizing the total inventory cost. Finally, the proposed model has been verified by a numerical example. In the future study, fuzzy concept can be applied for all provisions in this predictable model.

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